

Name: ANSWERS

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By writing my name above, I acknowledge complying with the CUNY Academic Integrity Policy while completing this examination.

MAT176 (Spring 2019)

Quiz 3

1. (5 pts) Evaluate the following improper integral:

$$\int_5^{+\infty} xe^{-x} dx = \lim_{b \rightarrow \infty} \int_5^b xe^{-x} dx = \lim_{b \rightarrow \infty} (-xe^{-x} - e^{-x}) \Big|_5^b$$

Parts

$$\int xe^{-x} dx = -xe^{-x} + \int e^{-x} = -xe^{-x} - e^{-x}$$

$$= \lim_{b \rightarrow \infty} -be^{-b} - e^{-b} - (-5e^{-5} - e^{-5}) = \frac{5}{e^5} + \frac{1}{e^5} = \boxed{\frac{6}{e^5}}$$

L'H
↓
L'Hospital ↓

$$\lim_{b \rightarrow \infty} \frac{b}{e^b} = \lim_{b \rightarrow \infty} \frac{1}{e^b} = 0.$$

2. (5 pts) Evaluate the following improper integral:

$$\int_0^{\pi/2} \tan(\theta) d\theta = \lim_{b \rightarrow \frac{\pi}{2}} \int_0^b \tan \theta d\theta = \lim_{b \rightarrow \frac{\pi}{2}} -\log|\cos \theta| \Big|_0^b$$

$$\int \tan \theta d\theta = \int \frac{\sin \theta}{\cos \theta} d\theta = -\int \frac{-\sin \theta}{\cos \theta} d\theta = -\log|\cos \theta| + C$$

$$= \lim_{b \rightarrow \frac{\pi}{2}} -\log|\cos b| + \log|1| = \boxed{+\infty} \text{ diverges}$$

" b

since $\lim_{b \rightarrow \pi/2} \cos b = \cos \frac{\pi}{2} = 0$ and $\lim_{y \rightarrow 0^+} -\log y = +\infty$.