Name: ANSWERS Lehman ID: _______________

By writing my name above, I acknowledge complying with the CUNY Academic Integrity Policy while completing this examination.

MAT176 (Spring 2019)
Quiz 4

Decide whether each of the following alternating series **converges or diverges**. **If it** converges, decide whether it **converges conditionally or absolutely**.

Justify your answers with appropriate convergence tests.

1. **(5 pts)** $\sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2+1}$ converges conditionally

Alternating Series Test: $a_n = \frac{n}{n^2+1}$ is positive, decreasing

b/c $f(x) = \frac{x}{x^2+1}$ has

$f'(x) = \frac{(x^2+1) - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} < 0$ if $x > 1$

and $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$

$\Rightarrow \boxed{\sum_{n=0}^{+\infty} \frac{(-1)^n n}{n^2+1} \text{ converges.}}$

To check if it converges absolutely:

$\sum_{n=1}^{\infty} \frac{n}{n^2+1} = +\infty$ diverges by Limit Comp. Test with $b_n = \frac{1}{n}$ (harmonic series)

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{n^2+1} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1 \Rightarrow \sum_{n=1}^{\infty} \frac{n}{n^2+1}$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ both behave in the same way (diverge).

Thus, $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ converges conditionally!

2. **(5 pts)** $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{(2n+1)!}$

Ratio test:

$L = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} \pi^{n+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{(-1)^n \pi^n} \right|$

$= \lim_{n\to\infty} \frac{\pi (2n+1)!}{(2n+3)!} = \lim_{n\to\infty} \frac{\pi \cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}}$

$= \lim_{n\to\infty} \frac{\pi}{(2n+3)(2n+2)} = 0 < 1$

Thus, this series converges absolutely!