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MAT176 (Spring 2019)
Quiz 5

Find the interval of convergence of the following power series. **Justify your answers** with appropriate convergence tests, and remember to check the endpoints!

1. **(5 pts)** $\sum_{n=1}^{\infty} \frac{4^n}{n}(x-3)^n$

(*) $\lim_{n\to\infty} n^{4/n} = \lim_{n\to\infty} e^{\frac{\ln n}{n}} = e^0 = 1.$

Root test: $L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{4}{n^{1/n}}|x-3| \overset{(*)}{=} 4|x-3| < 1$ ← need this for convergence

$3-\frac{1}{4}$ $\quad$ 3 $\quad$ $3+\frac{1}{4}$

Endpoints: $3+\frac{1}{4}$ (right): $\sum_{n=1}^{\infty} \frac{4^n}{n}\left(\frac{1}{4}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series) diverges

$3-\frac{1}{4}$ (left): $\sum_{n=1}^{\infty} \frac{4^n}{n}\left(-\frac{1}{4}\right)^n = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (Alt. harm. series) converges.

$\Rightarrow$ Interval of convergence is $I = \left[3-\frac{1}{4}, 3+\frac{1}{4}\right) = [2.75, 3.25)$

2. **(5 pts)** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n$

Root test: $L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{|x-1|}{n^{1/n}} \overset{(*)}{=} |x-1| < 1$ ← need this for convergence

0 $\quad$ 1 $\quad$ 2

Endpoints: 0 (left): $\sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n}(-1)^n = -\sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series) diverges

2 (right): $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ (Alt. harmonic series) converges

$\Rightarrow$ Interval of convergence is $I = (0, 2]$