Name: ANSWERS Lehman ID: ____________

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MAT176 (Spring 2019)
Quiz 6

1. **(8 pts)** Find the Maclaurin series of $f(x) = \dfrac{x}{(1+x)^2}$, using that $\dfrac{1}{1+x} = \sum_{n=0}^{+\infty}(-1)^n x^n$

$$\frac{d}{dx}\frac{1}{1+x} = \frac{d}{dx}\sum_{n=0}^{+\infty}(-1)^n x^n = \sum_{n=1}^{+\infty}(-1)^n n x^{n-1}$$

$$\parallel$$

$$-\frac{1}{(1+x)^2}$$

$$f(x) = \frac{x}{(1+x)^2} = -x\sum_{n=1}^{+\infty}(-1)^n n x^{n-1} = \sum_{n=1}^{+\infty}(-1)^{n+1} n x^n$$

So

$$\boxed{f(x) = \sum_{n=1}^{+\infty}(-1)^{n+1} n x^n}$$

2. **(2 pts)** Prove that $\sum_{n=1}^{+\infty}(-1)^{n+1}\dfrac{n}{2^n} = \dfrac{2}{9}$.

Hint: Compute $f(\frac{1}{2})$.

$$f\left(\tfrac{1}{2}\right) = \frac{\frac{1}{2}}{\left(\frac{3}{2}\right)^2} = \frac{1}{2}\,\frac{4}{9} = \frac{2}{9}$$

$$\parallel$$

$$f\left(\tfrac{1}{2}\right) = \sum_{n=1}^{+\infty}(-1)^{n+1} n\left(\tfrac{1}{2}\right)^n = \sum_{n=1}^{+\infty}(-1)^{n+1}\frac{n}{2^n}$$

$$\Rightarrow \boxed{\sum_{n=1}^{+\infty}(-1)^{n+1}\frac{n}{2^n} = \frac{2}{9}}$$