Solutions to HW 2

1. Equation of the plane through

$P=(1,2,3)$, $Q=(4,5,6)$, $R=(-1,0,-2)$

Need normal vector $\vec{n}=\vec{PQ}\times\vec{PR}$

$\vec{PQ}=Q-P=(3,3,3)$

$\vec{PR}=R-P=(-2,-2,-5)$

$$\vec{n}=\vec{PQ}\times\vec{PR}=\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 3 & 3 \\ -2 & -2 & -5 \end{vmatrix}$$

$$=(-15+6,-(-15+6),-6+6)$$

$$=(-9,9,0)$$

Equation of the plane is

$\langle (x,y,z)-P,\vec{n}\rangle=0$

$\langle (x-1,y-2,z-3),(-9,9,0)\rangle=0$

$-9(x-1)+9(y-2)=0$

$-9x+9+9y-18=0$

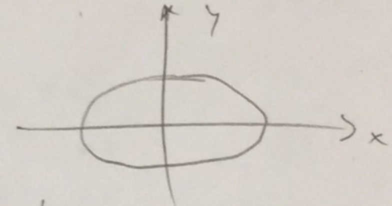
$-9x+9y-9=0$

$\boxed{-x+y-1=0}$

(or, equivalently, $x-y+1=0$)

2. $x^2 + 4y^2 = z^2$

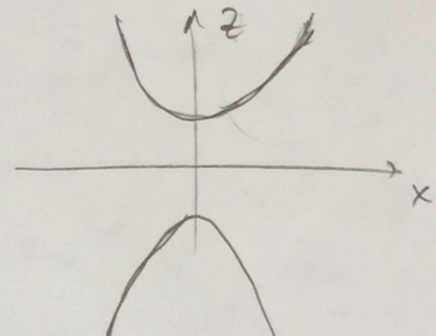
a) $z = z_0$: $x^2 + 4y^2 = z_0^2$ are ellipses ($\forall z_0 \neq 0$) in the xy-plane



b) $y = y_0$: $x^2 + 4y_0^2 = z^2$ are hyperbolas in the xz-plane

$$x^2 - z^2 = -4y_0^2$$

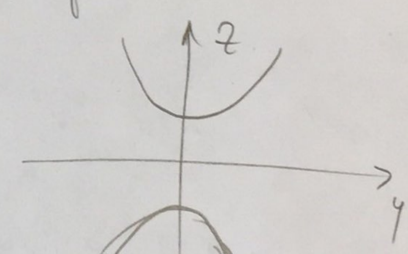
(and $|z| \geq 2|y_0| \geq 0$)



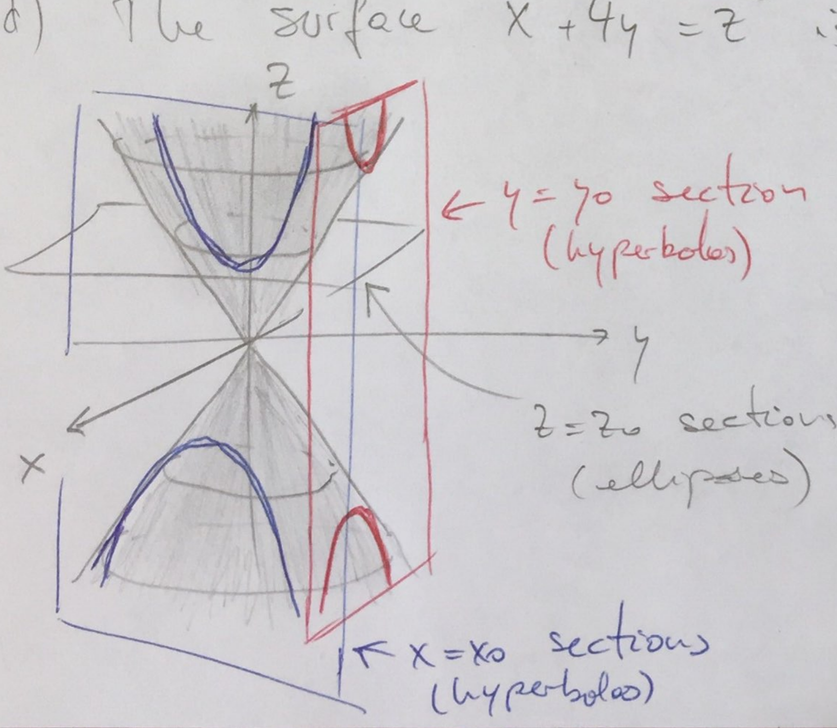
c) $x = x_0$: $x_0^2 + 4y^2 = z^2$ are hyperbolas in the yz-plane

$$4y^2 - z^2 = -x_0^2$$

(and $|z| \geq |x_0| \geq 0$)

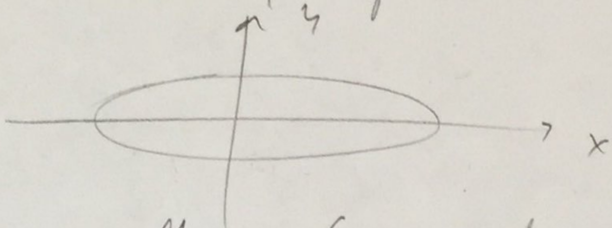


d) The surface $x^2 + 4y^2 = z^2$ is an elliptic cone

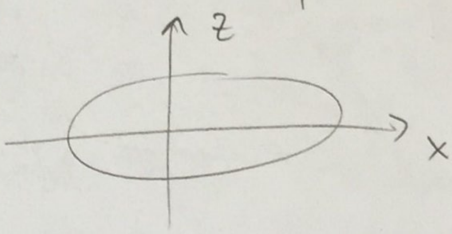


3. $x^2 + 4y^2 + z^2 = 1$

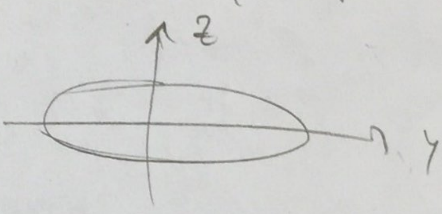
a) $z = z_0$: $x^2 + 4y^2 = 1 - z_0^2$ are <u>ellipses</u> (or empty) in the xy plane



b) $y = y_0$: $x^2 + z^2 = 1 - 4y_0^2$ are <u>ellipses</u> (or empty) in the xz-plane



c) $x = x_0$: $4y^2 + z^2 = 1 - x_0^2$ are <u>ellipses</u> (or empty) in the yz-plane



d) The surface $x^2 + 4y^2 + z^2 = 1$ is an <u>ellipsoid</u>