MAT226

Solutions to HW 6

1. $f(x,y) = x^4 - x^2y^2 + 3y + 8 \Rightarrow \begin{cases} \frac{\partial f}{\partial x}(x,y) = 4x^3 - 2xy^2 \\ \frac{\partial f}{\partial y}(x,y) = -2x^2y + 3 \end{cases}$

$F(x,y,z) = z - f(x,y)$

$\nabla F(x,y,z) = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right) = \left(-4x^3 + 2xy^2, 2x^2y + 3, 1\right)$

a) $(x_0,y_0) = (1,1)$ $\quad z_0 = f(x_0,y_0) = 1 - 1 + 3 + 8 = \underline{11}$

$\nabla F(1,1,11) = (-2,-1,1)$

<u>Tangent plane:</u> $\langle (x-x_0, y-y_0, z-z_0), \nabla F(x_0,y_0,z_0) \rangle = 0$

$\langle (x-1, y-1, z-11), (-2,-1,1) \rangle = 0$

$-2(x-1) - (y-1) + (z-11) = 0$

$\boxed{-2x - y + z - 8 = 0}$ (or $2x + y - z + 8 = 0$)

b) $(x_0,y_0) = (0,2)$ $\quad z_0 = f(x_0,y_0) = 3.2 + 8 = 14$

$\nabla F(0,2,14) = (0,-3,1)$

<u>Tangent plane:</u> $\langle (x-0, y-2, z-14), (0,-3,1) \rangle = 0$

$-3(y-2) + (z-14) = 0 \Rightarrow \boxed{-3y + z - 8 = 0}$ (or $3y - z + 8 = 0$)

c) $(x_0, y_0) = (1, -2)$ $\quad z_0 = f(x_0, y_0) = 1 - 4 - 6 + 8 = -1$

$\nabla F(1, -2, -1) = (4, -7, 1)$

tangent plane. $\langle (x-1, y+2, z+1), (4, -7, 1) \rangle = 0$

$4(x-1) - 7(y+2) + (z+1) = 0$

$\boxed{4x - 7y + z - 17 = 0}$ $\left(\text{or} \quad -4x + 7y - z + 17 = 0 \right)$