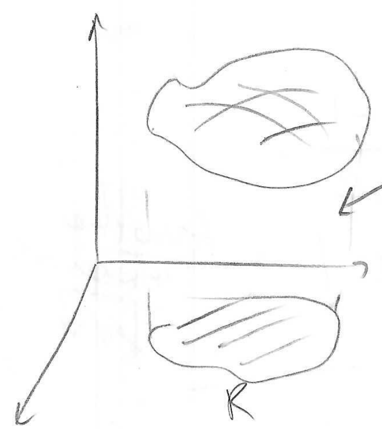
MAT226 Lecture 16 11/4/2019

Last time:

$z = f(x,y)$



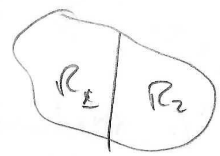
Volume in between graph of $f(x,y)$ and region R $= \iint_R f(x,y)\, dA$

Note: Also use for Area (take $f \equiv 1$)

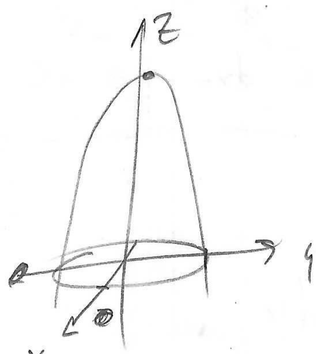
Properties: $\iint_R af + bg \; dA = a\iint_R f\,dA + b\iint_R g\, dA$ (linearity)

$$\iint_{R_1 \cup R_2} f\, dA = \iint_{R_1} f\,dA + \iint_{R_2} f\, dA.$$

disjoint R_1 & R_2



Ex: Find volume under paraboloid $z = 4 - x^2 - 2y^2$

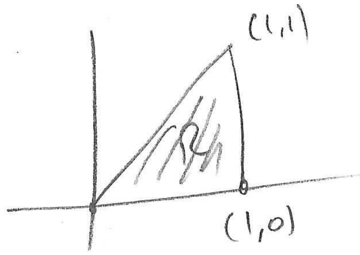


$$V = \int_{-2}^{2}\int_{-\sqrt{\frac{4-x^2}{2}}}^{\sqrt{\frac{4-x^2}{2}}} \left(4 - x^2 - 2y^2\right) dy\, dx$$

$$= \int_{-2}^{2} (4-x^2)y - \frac{2y^3}{3}\Big|_{-\sqrt{\ }}^{\sqrt{\ }} dx$$

$$= \frac{4}{3\sqrt{2}}\int_{-2}^{2}\left(4-x^2\right)^{3/2} dx \underset{x = 2\sin\theta}{=} \frac{4}{3\sqrt{2}}\int_{-\pi/2}^{\pi/2} 16\cos^4\theta\, d\theta = 4\sqrt{2}\pi$$

Find volume under $z = e^{-x^2}$ over region R



$$V = \int_0^1 \int_0^x e^{-x^2}\, dy\, dx = \int_0^1 x e^{-x^2}\, dx$$

$$= -\frac{1}{2} e^{-x^2} \Big|_0^1 = \frac{e-1}{2e}.$$

Note: Can't integrate in x first!

Another example first $\triangle$

Applications: • Average $= \frac{1}{\text{Area}(R)} \iint_R f(x,y)\, dA$

• Joint probability distribution

X, Y random var, "jointly continuous"

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y)\, dx\, dy = 1$$

$$P((X,Y) \in A) = \iint_A f(x,y)\, dx\, dy$$

Later!

Improper integrals: $R = \{(x,y) : x \geq 0,\ y \geq 0\}$

$$\boxed{\int x e^{-x^2} dx = -\frac{1}{2} e^{-x^2} + C}$$

$$\int_0^{+\infty} \int_0^{+\infty} xy e^{-x^2-y^2}\, dx\, dy = ?$$

$$\int_0^b \int_0^a xy e^{-x^2-y^2}\, dx\, dy = \int_0^b y \left(-\frac{1}{2} e^{-x^2-y^2} \right) \Big|_0^a dy$$

$$= \int_0^b -\frac{1}{2} y e^{-a^2-y^2} + \frac{1}{2} y e^{-y^2}\, dy = \frac{1}{4} e^{-a^2-y^2} \Big|_0^b - \frac{1}{4} e^{-y^2} \Big|_0^b =$$

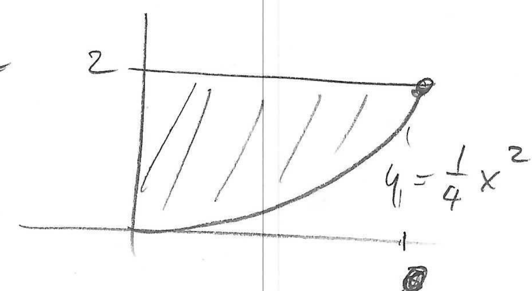
$$= \frac{1}{4} e^{-a^2-b^2}\left(e^{a^2}-1\right)\left(e^{b^2}-1\right) = \frac{1}{4} - \frac{e^{-a^2}}{4} - \frac{e^{-b^2}}{4} + \frac{1}{4}e^{-a^2-b^2}$$

$$\iint_R f(x,y)\,dx\,dy = \lim_{(a,b)\to(\infty,\infty)} \frac{1}{4} - \underbrace{\frac{e^{-a^2}}{4} - \frac{e^{-b^2}}{4} + \frac{1}{4}e^{-a^2-b^2}}_{\to 0} = \boxed{\frac{1}{4}}$$

Before:

$$\iint_R 5x^3\cos(y^3)\,dA$$

$R =$



$$= \int_0^2 \int_0^{2\sqrt{y}} 5x^3\cos(y^3)\,dx\,dy$$

$$= \int_0^2 \frac{5}{4}x^4\cos(y^3)\Big|_0^{2\sqrt{y}}\,dy = \int_0^2 20y^2\cos(y^3)\,dy =$$

$$= \frac{20}{3}\sin(y^3)\Big|_0^2 = \frac{20}{3}\sin(8) \cong \underline{6.5957}$$

(!) only 1 order of integration is feasible

$$\int \cos(y^3)\,dy = ?$$