MAT 330

Solutions to HW 2

#1 20 floors above ground

17 sequences of 4 consecutive floors:

17 possibilities:
$$\left\{\begin{array}{cccc} \underline{1} & \underline{2} & \underline{3} & \underline{4} \\ \underline{2} & \underline{3} & \underline{4} & \underline{5} \\ & \vdots & & \\ \underline{16} & \underline{17} & \underline{18} & \underline{19} \\ \underline{17} & \underline{18} & \underline{19} & \underline{20} \end{array}\right.$$

E = 4 people choose 4 consecutive floors

(17 possible 4-tuples of consecutive floors) $\cdot$ (4! reorderings)

$$P(E) = \frac{|E|}{|S|} = \boxed{\frac{17 \cdot 4!}{20^4}} \cong 0.00255 = 0.255\%$$

(20^4 total different sets with 4 out of 20 floors (w/ possible repetition))

#2 a) 4 sets of 13 choices correspond to entire suit

$$\boxed{\frac{4}{\binom{52}{13}}}$$

number of hands possible.

b) Let E_i = i^{th} suit is missing from your hand.

Inclusion-Exclusion principle

$$P(E_1 \cup E_2 \cup E_3 \cup E_4) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$$
$$- P(E_1E_2) - P(E_1E_3) - P(E_1E_4)$$
$$- P(E_2E_3) - P(E_2E_4) - P(E_3E_4)$$
$$+ P(E_1E_2E_3) + P(E_1E_2E_4) + P(E_1E_3E_4)$$
$$+ P(E_2E_3E_4) - P(E_1E_2E_3E_4)$$

$$= 4\frac{\binom{39}{13}}{\binom{52}{13}} - 6\frac{\binom{26}{13}}{\binom{52}{13}} + \frac{4}{\binom{52}{13}} = \boxed{\frac{4\binom{39}{13} - 6\binom{26}{13} + 4}{\binom{52}{13}}}$$

Note: $P(E_i) = \frac{\binom{39}{13}}{\binom{52}{13}}$, $52-13=39$ cards to choose 13 from

$P(E_iE_j) = \frac{\binom{26}{13}}{\binom{52}{13}}$, $52-2 \cdot 13 = 26$ cards to choose 13 from

$P(E_iE_jE_k) = \frac{1}{\binom{52}{13}}$, $P(E_1E_2E_3E_4) = 0$