MAT 330/681 Solutions to HW 5

#1. E = 4 heads appear (evidence)

H = we are using the trick coin (hypothesis)

$$P(H|E) \overset{\text{Bayes}}{=} \frac{P(E|H)\,P(H)}{P(E|H)\,P(H) + P(E|H^c)\,P(H^c)}$$

$$= \frac{1 \cdot 1/5}{1 \cdot 1/5 + \left(1/2\right)^4 \cdot 4/5} = \frac{1}{1 + 4/2^4}$$

$$= \frac{1}{1 + 1/4} = \frac{1}{5/4} = \boxed{\frac{4}{5}} \quad (= 80\%)$$

#2. This is the Gambler Ruin Problem, with

$i = 10$, $N = 20$, $p = \frac{4}{10}$, $q = \frac{6}{10}$, for which:

$$P(\text{win}) = \frac{1-\left(\frac{q}{p}\right)^i}{1-\left(\frac{q}{p}\right)^N} = \frac{1-\left(\frac{3}{2}\right)^{10}}{1-\left(\frac{3}{2}\right)^{20}}$$

$$P\left(\substack{\text{run out}\\\text{of money}}\right) = 1 - P(\text{win}) = \boxed{1 - \frac{1-\left(\frac{3}{2}\right)^{10}}{1-\left(\frac{3}{2}\right)^{20}} = \frac{\left(\frac{3}{2}\right)^{10} - \left(\frac{3}{2}\right)^{20}}{1-\left(\frac{3}{2}\right)^{20}}}$$

$$\left(= \frac{1024}{60073} \cong 98.3\%\right)$$