MAT330

Solution to HW 7

#1. X = # defective products in random batch of 10 products

$X \sim \text{Binomial}\left(10, \frac{1}{10}\right)$

$n = 10,\ p = \frac{1}{10}$ (product is defective)

a) $P(X=0) = \binom{10}{0} p^0 (1-p)^{10} = \left(\frac{9}{10}\right)^{10} \cong \boxed{0.3487}$ $(\cong 34.87\%)$

b) $P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - \binom{10}{0} p^0 (1-p)^{10} - \binom{10}{1} p^1 (1-p)^9 - \binom{10}{2} p^2 (1-p)^8$

$= 1 - \left(\frac{9}{10}\right)^{10} - 10 \frac{1}{10}\left(\frac{9}{10}\right)^9 - 45 \frac{1}{10^2}\left(\frac{9}{10}\right)^8$

$= \dfrac{10^{10} - 9^{10} - 10 \cdot 9^9 - 45 \cdot 9^8}{10^{10}} \cong \boxed{0.0702}$ $(\cong 7.02\%)$

#2 X = # phone calls that arrive to President's office in each 2min period

$X \sim \text{Poisson}(\lambda)$,

$7 = \overset{\text{outside}}{3} + \overset{\text{inside}}{4}$ expected calls/10 min correspond to 7/5 calls in each 2 min period.

$$\lambda = E(X) = \frac{7}{5}$$

Thus $P(X=i) = \dfrac{e^{-\frac{7}{5}}\left(\frac{7}{5}\right)^i}{i!}$ $\quad \forall i = 0, 1, 2, \ldots$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

$$= 1 - \frac{e^{-\frac{7}{5}}\left(\frac{7}{5}\right)^0}{0!} - \frac{e^{-\frac{7}{5}}\left(\frac{7}{5}\right)^1}{1!}$$

$$= 1 - e^{-\frac{7}{5}}\left(1 + \frac{7}{5}\right) = \boxed{1 - \frac{12}{5} e^{-\frac{7}{5}}}$$

$$\simeq 0.4081$$

$$(\simeq 40.81\,\%)$$