MAT 330

Solution to HW9

#1. $X \sim \mathrm{Unif}([0,1])$, $Y = e^X$

$F_X(x) = x$

$$F_Y(y) = P(Y \le y) = P(e^X \le y) = P(X \le \ln y) = \ln y$$

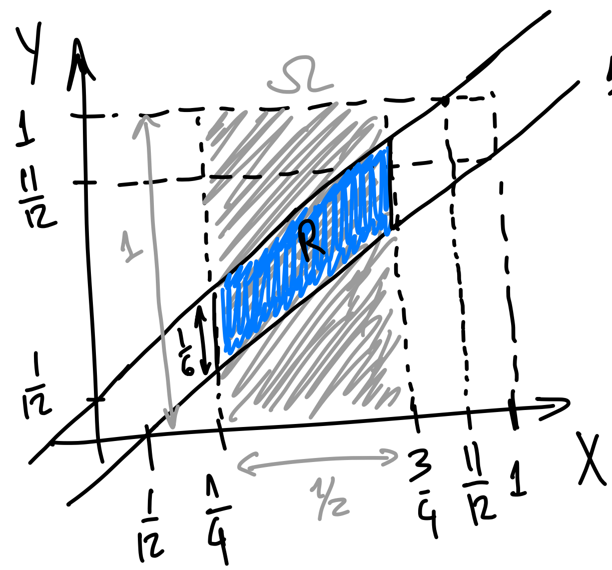
$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{d}{dy} \ln y = \frac{1}{y} \quad \text{if} \quad y \in [1, e]$$

$$\boxed{f_Y(y) = \begin{cases} \frac{1}{y} & \text{if } y \in [1,e] \\ 0 & \text{otherwise} \end{cases}}$$

2. $$\int_{-\infty}^{+\infty} f_Y(y)\,dy = \int_1^e \frac{1}{y}\,dy = \ln y \Big|_1^e = \ln e - \ln 1 \stackrel{!}{=} \boxed{1}$$

3. No, Y is not distributed as an exponential rand. var.

#2. As fractions of an hour: $X \sim \mathrm{Unif}\left(\left[\overset{3{:}15\text{pm}}{\tfrac{1}{4}}, \overset{3{:}45\text{pm}}{\tfrac{3}{4}}\right]\right)$, $Y \sim \mathrm{Unif}\left(\left[\overset{3{:}00\text{pm}}{0}, \overset{4{:}00\text{pm}}{1}\right]\right)$

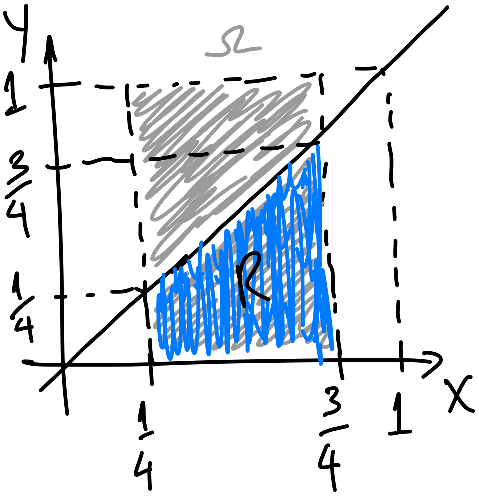


1. $$P\left(|X-Y| < \underset{5\text{ min}}{\frac{5}{60}}\right) = P\left(-\frac{1}{12} < X - Y < \frac{1}{12}\right)$$

$$= P\left(X - \frac{1}{12} < Y < X + \frac{1}{12}\right)$$

$$= \frac{\mathrm{Area}(R)}{\mathrm{Area}(\Omega)} = \frac{\frac{1}{6}\cdot\frac{1}{2}}{\frac{1}{2}\cdot 1} = \boxed{\frac{1}{6}}$$

2. $P(X<Y) = \dfrac{\text{Area}(R)}{\text{Area}(\Omega)} = \dfrac{\frac{1}{2}\cdot\frac{1}{2}\cdot 1}{\frac{1}{2}\cdot 1} = \boxed{\frac{1}{2}}$



Note: Could also solve with integrals (of constant functions) but that is unnecessary here; can simply use the areas of the corresponding regions.