Name: ANSWERS Lehman ID: _____

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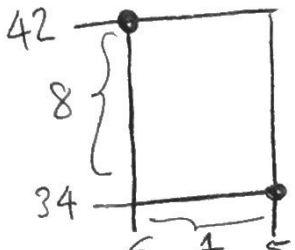
MAT 330/681 (Spring 2020)
Quiz 1

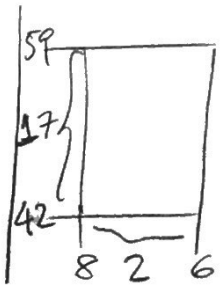
1. This problem builds on the HW1 problem about walking from the CUNY Graduate Center (located on the corner of 34th Street and 5th Avenue) to Columbus Circle (located on the corner of 59th Street and 8th Avenue). Just like in HW1, assume you are walking only along streets (moving west) and avenues (moving north) and not through buildings or on diagonal streets (such as Broadway), and also not taking unnecessary detours (that is, you only walk west and north, never east or south).

   a) **(3 pts)** Suppose your favorite candy store is on the corner of 42nd Street and 6th Avenue. How many ways are there to walk from CUNY GC to Columbus Circle *that pass by this candy store*?



1 block West
8 blocks North

$$\binom{9}{1} = 9$$



2 blocks West
17 blocks North

$$\binom{19}{2} = \frac{19!}{17!\,2!} = 171$$

   b) **(3 pts)** What is the probability of passing by the candy store if you choose at random a path from CUNY GC to Columbus Circle (assuming equal likelihood for each such path)?

total: a)

$$\binom{9}{1}\cdot\binom{19}{2} = 1539$$

b)

$$P(E) = \frac{|E|}{|S|} = \frac{\binom{9}{1}\binom{19}{2}}{\binom{28}{3}} = \cdots = \frac{171}{364} \cong 0.46978$$

   c) **(4 pts)** Your second favorite candy store is on the corner of 50th Street and 7th Avenue. What is the probability of passing by *at least one* among your favorite and second favorite candy stores if you choose at random a path from CUNY GC to Columbus Circle (again, assuming equal likelihood for each such path)?

A = Passing by 42nd and 6th: $P(A) = \dfrac{\binom{9}{1}\binom{19}{2}}{\binom{28}{3}}$

B = Passing by 50th and 7th: $P(B) = \dfrac{\binom{16}{2}\binom{12}{1}}{\binom{28}{3}}$

$A \cap B$ = Passing by both 42nd and 6th and 50th and 7th: $P(A\cap B) = \dfrac{\binom{9}{1}\binom{9}{1}\binom{10}{1}}{\binom{28}{3}}$

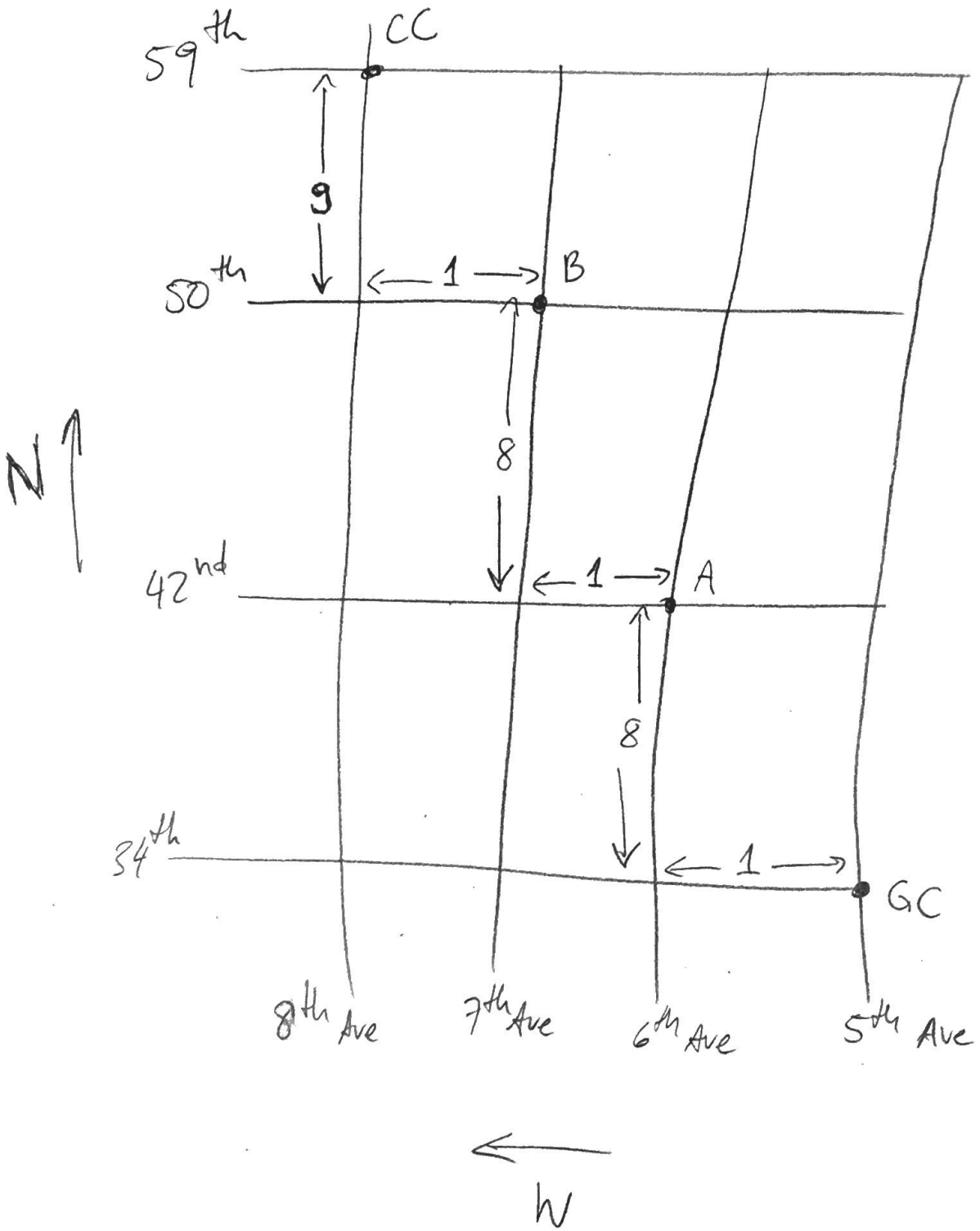
$A \cup B$ = Passing by at least one of the two candy shops.

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

c)

$$= \frac{\binom{9}{1}\binom{19}{2} + \binom{16}{2}\binom{12}{1} - \binom{9}{1}\binom{9}{1}\binom{10}{1}}{\binom{28}{3}}$$

$= 241/364 \cong 0.66208$


59th
CC
9
50th
1
B
8
42nd
1
A
8
34th
1
GC
N
8th Ave
7th Ave
6th Ave
5th Ave
W