MAT 330/681

Review for Midterm Exam

#1. How many permutations of the word CORONAVIRUS ?

11 letters
2 O's
2 R's

$$\binom{11}{2,2} = \frac{11!}{2!\,2!}$$

$11!$ ← Permutations of 11 symbols

$2!\,2!$ ← 2 of which are alike (O)
2 ————— " ————— (R)

In how many of them do the 2 O's appear together?

$\underbrace{(OO)}_{\text{1 combo symbol}}$CRNAVIRUS

$$\overset{\text{10 symbols}}{\frac{10!}{\underset{\text{2 R's}}{2!}}} \cdot \frac{\overset{\text{subpermutations of (OO)}}{\cancel{2!}}}{\underset{\text{2 O's}}{\cancel{2!}}} = 10!/2!$$

What's the prob. of selecting a permutation where both O's appear together?

$$P = \frac{10!/2!}{11!/2!2!} = \frac{10!}{11!} \cdot 2! = \boxed{\frac{2}{11}}$$

#2. What is the prob of getting a 6 in 5 straight tosses of a die?

$$\left(\frac{1}{6}\right)^5$$

What is the prob. that a 6 appears on the 6^{th} toss?

$\frac{1}{6}$ (independent events)

die tosses have no memory!

#3

Neighbor waters your plant while you're away
w/o water, it will die w/ 80% chance
w/ water, ——— " ——— 15% chance
You're 90% sure neighbor waters it.

a) What's the prob plant will survive?

S = Plant survives
W = Plant is watered

$$P(S) = P(S|W)P(W) + P(S|W^c)P(W^c)$$
$$= 0.85 \cdot 0.9 + 0.2 \cdot 0.1$$
$$= \boxed{0.785} \; (\simeq 78.5\%)$$

b) If plant dies, what's the prob your neighbor forgot to water it?

$$P(W^c|S^c) = \frac{P(S^c|W^c)\,P(W^c)}{P(S^c)} = \frac{0.8 \cdot 0.1}{0.215} = \boxed{\frac{16}{43}}$$

#4. If $P(A|B) = 1$, show $P(B^c|A^c) = 1$.

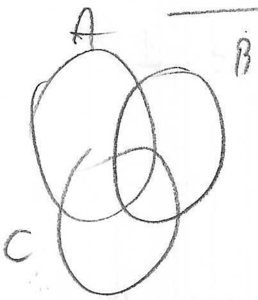
$$P(B^c|A^c) = \frac{P(A^cB^c)}{P(A^c)}$$

$$1 = P(A|B) = \frac{P(AB)}{P(B)}$$
$$\therefore P(B) = P(AB)$$

$$P(A^cB^c) = P((A\cup B)^c) = 1 - P(A\cup B)$$
$$= 1 - (P(A) + \cancel{P(B)} - \cancel{P(AB)}) = 1 - P(A) = P(A^c)$$

hence $P(B^c|A^c) = P(A^cB^c)/P(A^c) = P(A^c)/P(A^c) = 1.$

#5 Suppose that A, B, C are <u>independent</u> and <u>not mutually exclusive</u>. If $\begin{cases} P(A) = p \\ P(B) = q \\ P(C) = r \end{cases}$



find a formula for $P(A \cup B \cup C)$.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$$

$$= p + q + r - pq - pr - qr + pqr$$

$$\left(= \sigma_1 - \sigma_2 + \sigma_3\right) \quad \ldots ?$$

#6 Show that $\dfrac{P(H|E)}{P(G|E)} = \dfrac{P(H)}{P(G)} \dfrac{P(E|H)}{P(E|G)}$

E = evidence
H = hypothesis 1
G = hypothesis 2

$$\frac{P(H|E)}{P(G|E)} = \frac{\dfrac{P(HE)}{\cancel{P(E)}}}{\dfrac{P(GE)}{\cancel{P(E)}}} = \frac{P(HE)}{P(GE)} = \frac{P(E|H)\,P(H)}{P(E|G)\,P(G)}.$$

e.g., Q: before new evidence is observed $\dfrac{P(H)}{P(G)} = 3$

i.e. H is 3x more likely than G. If the evidence E is 2x more likely when G holds than when H holds, which hyp. is more likely after E is observed?

A: <u>H is 1.5x more likely</u>: $\dfrac{P(H|E)}{P(G|E)} = 3 \cdot \dfrac{P(E|H)}{P(E|G)} = \dfrac{3}{2} = 1.5$

#7 A store insurance policy costs \$5k, but would save an estimated \$250k in case of a theft. The likelihood of a theft is 1%. Should the store buy the insurance?

X = savings incurred by store if insurance is purchased. (in thousands of dollars)

$P(X=-5) = 0.99$, $P(X=245) = 0.01$

(no theft) (theft)

$E(X) = (-5) \cdot 0.99 + 0.01 \cdot 245 = -2.5 < 0.$

Should **not** buy insurance

#8 Gambler's Ruin Problem