MAT 330/681 Extra Review Problems 3/16/2020

#1

Q: How many words can be written with the letters in RESILIENCE?

A: 10 letters, 3 E's, 2 I's $\Big\}$ $\binom{10}{3,2} = \boxed{\dfrac{10!}{3!\,2!}}$

Q: How about if all vowels must appear together?

A: (EEEII) R S L N C

6 symbols

5 symbols, 3 E's, 2 I's

$$6! \cdot \binom{5}{3,2} = \boxed{6!\,\frac{5!}{3!\,2!}}$$

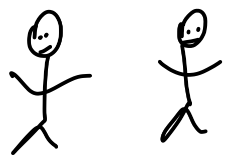
Q: What is the prob. that pick one w/ all vowels together?

A: $P = \dfrac{6!\;5!/3!2!}{\dfrac{10!}{3!2!}}$ ← words w/ all vowels together; ← total number of words

$$= \frac{6!5!}{\cancel{3!2!}} \cdot \frac{\cancel{3!2!}}{10!} = \boxed{\frac{6!5!}{10!}} = \boxed{\frac{1}{42}}$$

(simplifying)

#2



A B

Ⓐ: Lies 1/2 of the time

Ⓑ: Lies 2/3 of the time

$$P(L|A) = \frac{1}{2}, \quad P(L|B) = \frac{2}{3}$$

Q: What is the prob. that you're talking to <u>A</u> if he just told you a lie?

$$P(A) = P(B) = \frac{1}{2}$$

L: a lie happened.

$$P(A|L) = \frac{P(AL)}{P(L)} = \frac{P(L|A)P(A)}{P(L|A)P(A) + P(L|B)P(B)}$$

$$= \frac{1/2 \cdot \cancel{1/2}}{1/2 \cdot \cancel{1/2} + 2/3 \cdot \cancel{1/2}} = \frac{1/2}{\frac{1}{2} + \frac{2}{3}}$$

$$= \frac{1/2}{\frac{3+4}{6}} = \frac{1}{2} \cdot \frac{6}{7} = \boxed{\frac{3}{7}} \quad \left(< \frac{1}{2} \right)$$

$$P(B|L) = 1 - P(A|L) = 1 - \frac{3}{7} = \boxed{\frac{4}{7}}$$

$$P(B|L) = \frac{P(L|B)P(B)}{P(L|B)P(B) + P(L|A)P(A)} = \frac{2/3 \cdot \cancel{1/2}}{2/3 \cdot \cancel{1/2} + 1/2 \cdot \cancel{1/2}}$$

$$= \frac{2/3}{\frac{2}{3} + \frac{1}{2}} = \frac{2/3}{\frac{7}{6}} = \frac{2}{3} \cdot \frac{6}{7} = \boxed{\frac{4}{7}}$$

\#3 Netflix costs \$10/month

Download movie illegally and are caught fine \$125 for each movie.

Want to watch 3 movies in 1 month.

Prob. of being caught is $p = 20\% = \frac{1}{5}$.

What is estimated to be cheaper?

A: X = \$ spent if we download illegally

If $E(X) > 10$, then purchase Netflix

$E(X) < 10$, then download illegally

$$E(X) = \sum_x x \cdot P(X = x)$$

$$P(X=0) = (1-p)^3 = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$$

$\underset{(1)}{\underline{\qquad}}\ \underset{(2)}{\underline{\qquad}}\ \underset{(3)}{\underline{\qquad}}$

$$P(X=125) = 3(1-p)^2 \cdot p$$

$$= 3\left(\frac{4}{5}\right)^2 \cdot \frac{1}{5} = \frac{48}{125}$$

$$P(X=250) = 3(1-p)p^2 = 3\,\frac{4}{5} \cdot \left(\frac{1}{5}\right)^2 = \frac{12}{125}$$

$$P(X=375) = p^3 = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$$

$$E(X) = 0 \cdot P(X=0) + 125\, P(X=125) + 250\, P(X=250) + 375\, P(X=375)$$

$$= 0 + \cancel{125} \cdot \frac{48}{\cancel{125}} + 250 \cdot \frac{12}{125} + 375 \cdot \frac{1}{125}$$

$$= 48 + 2 \cdot 12 + 3 = 48 + 27 = \boxed{75}$$

Since $E(X) > 10$, it is expected that buying Netflix is cheaper. ☺

Post-video observation: We can also treat each illegal download individually, which has expected cost $\underset{\text{fine}}{125} \cdot \underset{\text{prob. of being caught}}{\frac{1}{5}} = 25$. If we do it 3 times, the expected cost is therefore $3 \cdot 25 = \boxed{75}$ as we found before.

This works because $E(\cdot)$ is linear, and more on this later!