MAT320/640, Fall 2021 **Renato Ghini Bettiol**

Homework Set 2

DUE: SEP 27, 2021 (VIA BLACKBOARD, BY 11.59PM)

To be handed in:

Please remember that all problems will be graded!

1. [Ross, Exercises 4.1-4.4] Answer the following questions about each of the sets:

$$A = (-1,1), \qquad B = \{\pi, e\}, \qquad C = \left\{\tfrac{1}{n} : n \in \mathbb{N} \text{ and } n \text{ is prime}\right\},$$
$$D = \{x^2 : x \in \mathbb{R}\}, \quad E = \bigcap_{n=1}^{\infty} \left[-\tfrac{1}{n}, 1+\tfrac{1}{n}\right], \quad F = \left\{\sin\left(\tfrac{n\pi}{3}\right) : n \in \mathbb{N}\right\}.$$

   (i) Is it bounded from below? (If so, exhibit an explicit lower bound.)

   (ii) Is it bounded from above? (If so, exhibit an explicit upper bound.)

   (iii) Compute its infimum.

   (iv) Compute its supremum.

   Recall that $\inf S = -\infty$ *if* S *is unbounded from below; and* $\sup S = +\infty$ *if* S *is unbounded from above, as we chose to convention in Video 4 of Lecture 4.*

2. [Ross, Exercises 4.7(a), 5.6] Let $S, T \subset \mathbb{R}$ be nonempty subsets of $\mathbb{R}$, such that $S \subset T$. Prove that

$$\inf T \leq \inf S \leq \sup S \leq \sup T.$$

   Give concrete examples of sets S and T to show that some (which?) inequalities above might be equalities even if $S \subsetneq T$, i.e., even if S and T do not coincide.