

## Homework Set 7

DUE: DEC 6, 2021 (VIA BLACKBOARD, BY 11.59PM)

**To be handed in:***Please remember that all problems will be graded!*

1. Prove that  $|\ln x - \ln y| \leq 5|x - y|$  for all  $x, y \in [\frac{1}{5}, 5]$ .
2. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a smooth function such that  $f(0) = 3$  and all derivatives of  $f(x)$  vanish at  $x = 0$ , that is,  $f'(0) = f''(0) = \dots = f^{(n)}(0) = \dots = 0$  for all  $n \in \mathbb{N}$ . Does there exist  $\varepsilon > 0$  such that  $f(x) = 3$  for all  $x \in (-\varepsilon, \varepsilon)$ ?
3. Consider the function  $f: [0, 1] \rightarrow \mathbb{R}$  given by

$$f(x) = \begin{cases} x, & \text{if } x \in \mathbb{Q}, \\ 0, & \text{if } x \notin \mathbb{Q} \end{cases}$$

- (a) Compute explicitly the lower and upper (Darboux) integrals of  $f(x)$  on the interval  $[0, 1]$ , that is, find the values of  $U(f)$  and  $L(f)$ .
- (b) Is  $f(x)$  integrable on  $[0, 1]$ ?