MAT 330/681 Lecture 28 5/17/2021

Review 2

Reminders:

- SETL Survey
- COVID Vaccine
- Final Exam info
  - Covers Lectures 1-23
  - 16 questions ← All of them are similar to problems from past HW assignments and lecture exercises
  - Reloads with new constants
  - Max. allowed time is 3 hours
  - Available from 12:00am - 11:59pm on May 19
  - Multiple attempts possible, need to resubmit all answers (even the ones that were correct)
    Grade is the average of attempts.

Problems to review:

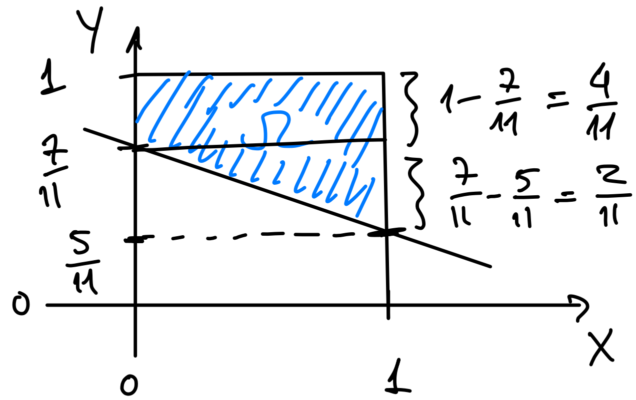
"Too much" for the Final:
- matching problem
- gambler's ruin problem
- menage problem.
- Monty Hall problem

- HW9: #~~1~~, ~~2~~
- HW2: #3 — see Lecture 27
- HW3 / HW8 / HW12 — #3
- HW12 #2 — #2
- HW6 #3

HW9 #1

$$X, Y \sim \text{Uniform}((0,1))$$

$$P(\underbrace{2X + 11Y - 7 > 0}_{\Omega}) = \frac{\text{Area}(\Omega)}{\text{Area(square)}} \rightarrow 1$$



$$P(2X+11Y-7>0) = P(11Y > 7-2X) = P\left(Y > \frac{7}{11} - \frac{2}{11}X\right)$$

$$\text{Area}(\Omega) = \frac{4}{11} + \frac{\cancel{2}}{11}\cdot\frac{1}{\cancel{2}} = \frac{5}{11}$$

HW9 #2.

$$X \sim \text{Exponential}(\underbrace{1/2}_{\lambda}) \qquad \boxed{X \geq 0}$$

p.d.f. of Y

$$Y = 13X^2 + 8 \qquad f_Y(10) = ?$$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases} \rightsquigarrow F(x) = \underbrace{P(X \leq x) = 1 - e^{-\lambda x}}$$

$$F_Y(y) = P(Y \leq y) = P(13X^2 + 8 \leq y) = P(13X^2 \leq y - 8)$$

$$= P\left(X^2 \leq \frac{y-8}{13}\right) = \underbrace{P\left(X \leq \sqrt{\frac{y-8}{13}}\right) = 1 - e^{-\frac{1}{2}\sqrt{\frac{y-8}{13}}}}$$

$$f_Y(y) = \frac{d}{dy} F_Y(y) = +e^{-\frac{1}{2}\sqrt{\frac{y-8}{13}}} \cdot \left(+\frac{1}{2} \cdot \frac{1}{2}\left(\frac{y-8}{13}\right)^{-\frac{1}{2}} \cdot \frac{1}{13}\right)$$

$$= \frac{1}{52}\sqrt{\frac{13}{y-8}}\, e^{-\frac{1}{2}\sqrt{\frac{y-8}{13}}}$$

$$f_Y(10) = \frac{1}{52}\sqrt{\frac{13}{2}}\, e^{-\frac{1}{2}\sqrt{\frac{2}{13}}}$$

HW 6 #3

$X \sim \text{Binomial}(n, p)$

$n = 24$

$p = 0.13$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$E(X) = n \cdot p$

$\mathrm{Var}(X) = n \cdot p \cdot (1-p)$ $\qquad \mathrm{Var}(X) = E(X^2) - E(X)^2$

$\sigma(X) = \sqrt{\mathrm{Var}(X)} = \sqrt{n \cdot p \cdot (1-p)}$

Standard deviation of X is $\sqrt{24 \cdot 0.13 \cdot 0.87}$

HW 12 #1

X = # strawberries produced

$\mu_X = 466$ strawberries/week

$\sigma_X^2 = 739$

One-sided Chebyshev ineq.:

If Y is nonnegative and $E(Y)=0$, then

$$P(Y \geq a) \leq \frac{\sigma^2}{a^2 + \sigma^2}$$

$E(aX+b) = aE(X) + b$ $\qquad a=1$, $b=-\mu$

$E(X-\mu) = E(X) - \mu = 0.$

$Y = X - \mu$ $\leftarrow$ Apply Chebyshev here (since $E(Y)=0$.) $\qquad \mathrm{Var}(Y) = \mathrm{Var}(X-\mu) = \mathrm{Var}(X).$

$\sigma_Y = \sigma_X$

$$P(X \geq 563) = P(\underbrace{X - 466}_{Y} \geq \underbrace{563-466}_{97}) = P(Y \geq 97) \leq \frac{\sigma_Y^2}{97^2 + \sigma_Y^2}$$

$$= \frac{739}{97^2 + 739} \cong 0.073$$

#3 $X_1, \ldots, X_9$ iid $\qquad X_i \quad i = 1, \ldots, 9$

$\mu = 3, \quad \sigma = 5$

$E(X_i) = \mu = 3$

$Var(X_i) = \sigma^2 = 25$

$$X = \overline{X_9} = \frac{X_1 + \cdots + X_9}{9}$$

$E(aX + bY) = aE(X) + bE(Y)$

$$E(X) = E\left(\frac{X_1 + \cdots + X_9}{9}\right) = \frac{1}{9}\left(\underbrace{E(X_1)}_{=3} + \cdots + \underbrace{E(X_9)}_{=3}\right) = \frac{\cancel{9} \cdot 3}{\cancel{9}} = \mu.$$

X, Y independent: $Var(X+Y) = Var(X) + Var(Y)$

$Var(aX) = a^2 Var(X)$

$$Var(X) = Var\left(\frac{X_1 + \cdots + X_9}{9}\right) = \frac{1}{9^2}\left(\underbrace{Var(X_1)}_{=25} + \cdots + \underbrace{Var(X_9)}_{=25}\right) = \frac{\cancel{9} \cdot 25}{9^{\cancel{2}}} = \frac{\sigma^2}{9}$$

$$E(X) + Var(X) = 3 + \frac{25}{9}$$

Discrete Random Variables

p.m.f. = prob. mass function

$p(x) = P(X = x)$

Continuous Random Variables

p.d.f. = prob. density function

$f(x) \qquad P(a \leq X \leq b) = \int_a^b f(x)\,dx$