Lecture 1

Exercise 1. You're starting a granola company and need to create a mix using two ingredients (almonds and oats) that meets nutritional guidelines while minimizing costs. Each ounce of almonds contains 8g of protein and 4g of fiber. Each ounce of oats provides 6g of protein and 12g of fiber. To meet dietary requirements, each granola bag must contain at least 11g of protein and 16g of fiber. Almonds costs $1.00 per ounce, while oats costs $1.20 per ounce. How many ounces of almonds and oats should go in each granola bag to minimize costs while meeting the dietary requirements?

Solution to Exercise 1. From the statement of the problem we have:

	Almonds (1 oz)	Oats (1 oz)
Protein	8g	6g
Fiber	4g	12g

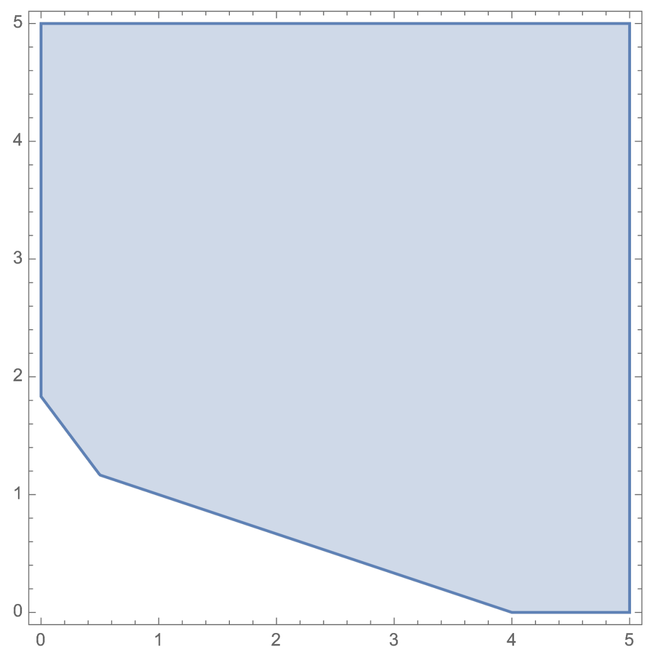
If we use x ounces of almonds and y ounces of oats, then the dietary constraints to be satisfied are

$$\begin{cases} 8x + 6y \geq 11 \\ 4x + 12y \geq 16 \\ x \geq 0, \ y \geq 0 \end{cases}$$

We want to find the minimum value of total cost $\$1.00\,x + \$1.20\,y$ among the points (x, y) satisfying the above constraints. In other words, we want to solve the optimization problem

$$\min \quad 1.00\,x + 1.20\,y \quad \text{s.t.} \quad \begin{aligned} 8x + 6y &\geq 11 \\ 4x + 12y &\geq 16 \\ x \geq 0, \ y &\geq 0 \end{aligned}$$

Plotting the *feasible region*, i.e., the (x, y) that satisfy these constraints, we obtain:



Overlapping the feasible region with plots of the levelsets $\{c(x, y) = t\}$ of the cost function $c(x, y) = 1.00\,x + 1.20\,y$, which are the black parallel lines shown in the next page, we see that the minimum must be achieved at the vertex $\left(\frac{1}{2}, \frac{7}{6}\right)$, where the cost is $c\left(\frac{1}{2}, \frac{7}{6}\right) = \frac{19}{10}$. The levelset $\{c(x, y) = \frac{19}{10}\}$ is shown in red. (The second picture just zooms in near the optimal solution, showing more levelsets, i.e., in "finer resolution".)

Thus, the optimal granola bag has to contain $\frac{1}{2}$ ounces of almonds and $\frac{7}{6}$ ounces of oats.