

## Homework Set 5

DUE: OCT 23-25, 2017 (AT THE BEGINNING OF RECITATION)

1. Consider the function  $f(x) = \begin{cases} 0, & x < 0 \\ Cx^3e^{-x}, & x \geq 0. \end{cases}$

- (a) What value of  $C$  makes the function  $f(x)$  a probability density?  
(b) What is the mean of this probability distribution?

2. Recall that the standard deviation of a random variable with probability density function  $f$  and mean  $\mu$  is given by

$$\sigma = \left( \int_{-\infty}^{+\infty} (x - \mu)^2 f(x) dx \right)^{1/2}.$$

Compute the standard deviation  $\sigma$  of a random variable with exponential probability density function with mean  $\mu$ .

3. Decide if the sequence  $\{a_n\}$  converges or diverges. If it converges, find its limit.

(a)  $a_n = \frac{1}{1+n^2}$

(b)  $a_n = \frac{n}{n+3}$

(c)  $a_n = \frac{1+n^2}{2+n}$

(d)  $a_n = 2^{1/n}$

(e)  $a_n = n!$

(f)  $a_n = \sqrt[n]{2n}$

(g)  $a_n = \frac{n!}{n^n}$

(h)  $a_n = \left( \frac{n}{n+1} \right)^n$