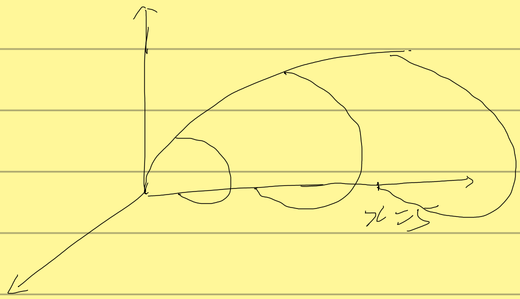
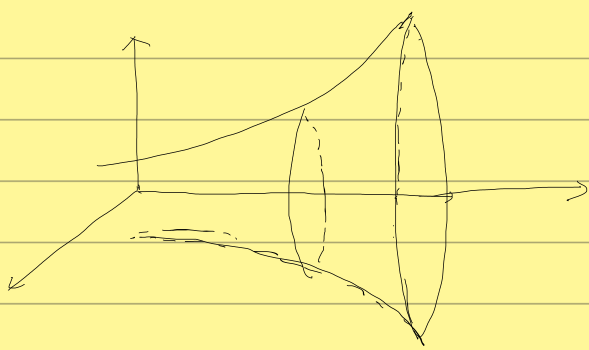
Solutions for HW1.

1.



Each cross-section is a half-disk,
with radius $r = \frac{2\sqrt{x}}{2} = \sqrt{x}$.
So the volumn is $\int_0^5 \frac{\pi}{2}(\sqrt{x})^2\,dx = \frac{25}{4}$

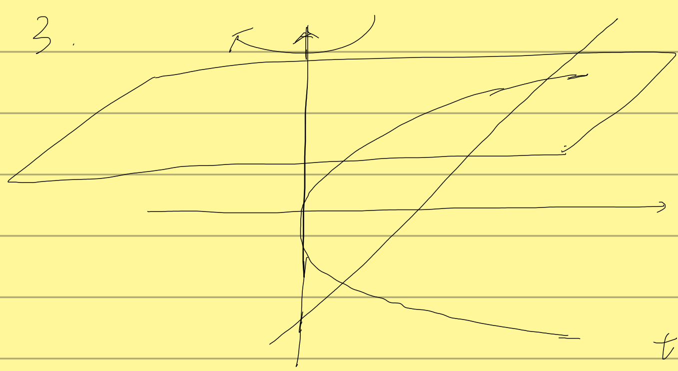
2.



The area of each cross-section is $\pi r^2 = \pi e^{2x}$.
thus the volumn is

$$\int_0^{\ln 2} \pi e^{2x}\,dx = \frac{\pi}{2} e^{2x}\Big|_0^{\ln 2} = \frac{\pi}{2}(4-1) = \frac{3\pi}{2}$$

3.



two intersection points are

$$\begin{cases} x = \frac{1}{2}y^2 \\ y = x-4 \end{cases} \Rightarrow \begin{cases} x_1 = 2, & x_2 = 8. \\ y_1 = -2 & y_2 = 4 \end{cases}$$

form our cross-section like this,
then the area of each cross-section is
$\pi\left[(y+4)^2 - (\frac{1}{2}y^2)^2\right]$

Thus the volumn is

$$\pi\int_{-2}^{4}\left[(y+4)^2 - (\tfrac{1}{2}y^2)^2\right]dy = \frac{576}{5}\pi$$

4. Use shell method, radius = x
height = $\sqrt{x} - \frac{1}{2}$.

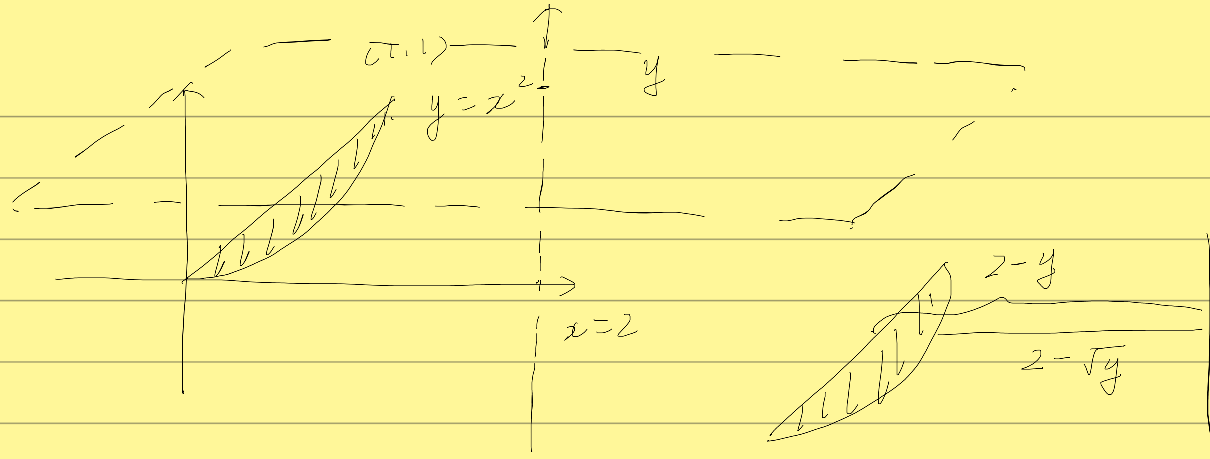
$$\int_1^3 2\pi x \cdot \left(\sqrt{x} - \frac{1}{2}\right) dx$$

$$= \int_1^3 \left(2\pi x^{\frac{3}{2}} - \pi x\right) dx = \pi\left(\frac{4}{5} x^{\frac{5}{2}} - \frac{x^2}{2}\right)\Bigg|_1^3$$

$$= \pi\left[\left(\frac{36}{5}\sqrt{3} - \frac{9}{2}\right) - \left(\frac{4}{5} - \frac{1}{2}\right)\right]$$

$$= \pi\left(\frac{36\sqrt{3}}{5} - \frac{24}{5}\right)$$

5.



Take y as my coordinate.

The area of the cross-section is

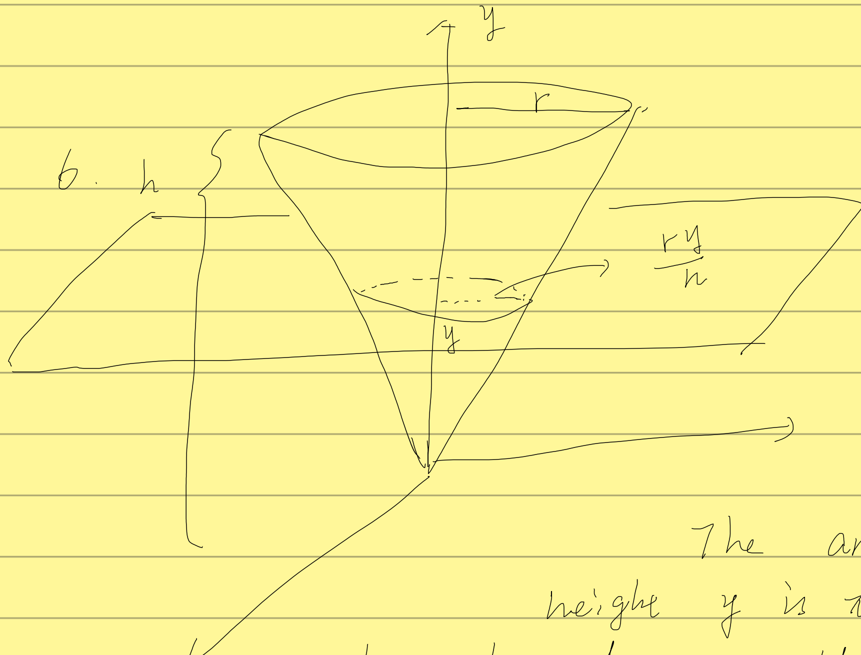
$$\pi\left((2-y)^2-(2-\sqrt{y})^2\right)$$

Thus the volumn is $\pi\int_0^1 \left((2-y)^2-(2-\sqrt{y})^2\right)dy$

$$=\pi\int_0^1\left[(4-4y+y^2)-(4-4\sqrt{y}+y)\right]dy$$

$$=\pi\left(\frac{y^3}{3}-\frac{5}{2}y^2+\frac{8}{3}y^{\frac{3}{2}}\right)\Big|_0^1=\frac{\pi}{2}$$

6.



The area of the cross-section at height y is $\pi(r\cdot\frac{y}{h})^2$.

Thus the volumn would be

$$\pi\int_0^h \frac{r^2y^2}{h^2}dy=\frac{\pi}{3}r^2h$$