Math 104, Spring 2018   **Renato Ghini Bettiol**

Homework Set 6

Due: Mar 14 - 16, 2018 (at the beginning of recitation)

1. Consider the function $f(x) = \begin{cases} 0, & x < 0 \\ Cx^2 e^{-2x}, & x \geq 0. \end{cases}$.

   (a) What value of C makes the function $f(x)$ a probability density?

   (b) What is the mean of this probability distribution?

2. Recall that the standard deviation of a random variable with probability density function f and mean μ is given by

$$\sigma = \left(\int_{-\infty}^{+\infty} (x - \mu)^2 f(x) \, \mathrm{d}x \right)^{1/2}.$$

   Compute the standard deviation σ of a random variable with exponential probability density function with mean μ.

3. Decide if the sequence $\{a_n\}$ converges or diverges. If it converges, find its limit.

   (a) $a_n = \dfrac{3}{2 + n^2}$

   (b) $a_n = \dfrac{2n}{5n + 7}$

   (c) $a_n = \dfrac{1 + n^2}{2 + 10n}$

   (d) $a_n = 3^{1/n}$

   (e) $a_n = n!$

   (f) $a_n = \sqrt[n]{4n}$

   (g) $a_n = \dfrac{n!}{n^n}$

   (h) $a_n = \left(\dfrac{n}{n + 1} \right)^n$