Math 104, Fall 2018

Practice Problems for Midterm 1

1. What is the volume of the solid obtained by revolving the region bounded by the line $y = 2x$ and the parabola $y = 2 - x^2$ about the line $x = 1$?

2. Find the volume of the solid obtained by revolving the region bounded between the x-axis and the curve $y = \cosh x$, $-2 \leq x \leq 2$, about the x-axis.
   HINT: Recall that $\cosh x = \dfrac{e^x + e^{-x}}{2}$.

3. Find the volume of the solid obtained by revolving about the x-axis the region bounded by the curve $y^2 = \cos x$ and the lines $y = 0$, $y = 1$, $x = -\frac{\pi}{2}$, and $x = \frac{\pi}{2}$.

4. Find the length of the curve $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}$ for $1 \leq x \leq \sqrt{2}$.

5. Find the length of the curve $y = \dfrac{e^x}{4} + \dfrac{1}{e^x}$ for $0 \leq x \leq 1$.

6. Use an integral to find the surface area of the (side of the) cone obtained by revolving the line $y = 2x$ about the x-axis, for $0 \leq x \leq 3$.

7. Use an integral to find the surface area of the (side of the) cone obtained by revolving the line $y = 2x$ about the y-axis, for $0 \leq x \leq 3$.

8. What is the centroid of the region bounded by the curves $y = e^x$ and $y = -1$ for $0 \leq x \leq \ln 2$?

9. Compute the following trigonometric integrals:

   a) $\displaystyle\int_0^{\pi/2} \sin(x)\cos^3(x)\,\mathrm{d}x$

   b) $\displaystyle\int_0^{\pi/2} \sin^2(x)\cos^3(x)\,\mathrm{d}x$

   c) $\displaystyle\int_0^{3\pi/2} \sin^3(x)\cos^3(x)\,\mathrm{d}x$

   d) $\displaystyle\int_0^{\pi/6} \tan^4(x)\sec^4(x)\,\mathrm{d}x$

10. Use integration by parts to compute the following indefinite integrals:

a) $\int x\, e^x \, \mathrm{d}x$

b) $\int e^{-x} \sin(x) \, \mathrm{d}x$

c) $\int x^2 \sin(4x) \, \mathrm{d}x$

d) $\int \ln(4x) \, \mathrm{d}x$

11. Use a trigonometric substitution to compute the following indefinite integrals:

a) $\int \frac{x}{\sqrt{9 - x^2}} \, \mathrm{d}x$

b) $\int \frac{1}{\sqrt{1 + x^2}} \, \mathrm{d}x$

c) $\int \sqrt{1 - 9x^2} \, \mathrm{d}x$

d) $\int \frac{1}{\sqrt{4x - x^2}} \, \mathrm{d}x$