## Math 110, Spring 2016 HWK01 due Jan 27

1. (a) What approximate value do you get for  $\sqrt{105}$  if you use a linear approximation to the function  $\sqrt{x}$  near x = 100? Now use a calculator to find a more precise value and state the absolute error of the approximation and the relative error (error as a proportion of the correct value).

(b) In the same manner, estimate 1/105. Say what approximation you are using, centered where, and give the absolute and relative error.

2. Compute the following limits. Please indicate the steps of your derivation. A hint to get started on the last one is to multiply and divide by  $\sqrt{x^2 + x} + x$ .

(a) 
$$\lim_{x \to 3} \frac{x^2 - 9}{x^2 - 4x + 3}$$

(b) 
$$\lim_{x \to 5^-} \frac{|2x - 10|}{x - 5}$$

(c) 
$$\lim_{x \to \infty} \frac{2x^2 + 3x + 4}{5x^2 + 6x + 7}$$

(d) 
$$\lim_{x \to \infty} \ln(3x) - \ln(x)$$

(e) 
$$\lim_{x \to \infty} \sqrt{x^2 + x} - x$$

3. Use L'Hôpital's rule, perhaps more than once, to evaluate these limits.

(a) 
$$\lim_{x \to 1^+} \frac{\ln x}{\sqrt{x-1}}$$

(b) 
$$\lim_{x \to \infty} \frac{x^3}{e^x}$$

4. Read the problem slowly and try to solve it. After 10 minutes, if you haven't solved it, go to the next page where the problem is broken down for you and solve it using the steps given. Understanding what constitutes a correct answer is really more of an issue of logic and notation than of computation.

Let f(A) be the time it takes for a machine to harvest an area A in a field of soybeans. Let g(A) be the time it takes a single worker to do this by hand. Do not assume f and g are linear, as there may be influences from learning, fatigue, and so forth. Do assume that workers and machines can join forces by working independently on different parts of the field. Write a formula for the amount of time it takes a machine plus five workers to harvest an area, A. The unspecified functions f and g will of course appear in the formula, as may one or more inverse functions.

Let's say we're measuring area in acres and time in hours.

- (a) How many acres can a worker harvest in an amount of time, t?
- (b) How many acres can a machine harvest in an amount of time, t?
- (c) How many acres can five workers and a machine harvest in an amount of time, t? This is where the part of the problem that begins "Do assume..." is important.
- (d) Name the function in the previous part: let L(t) be the number of acres that a machine and five workers can harvest in t hours. What function tells how many hours it takes for a machine and five workers to harvest A acres?
- (e) Write the answer to the question that was asked in a sentence: the number of hours it takes for a machine and five workers to harvest A acres is ... where L(t) is the function ...