Homework Set 7

DUE: MAR 16, 2017 (IN CLASS)

1. Compute the following multiple integrals:

   (a) $\displaystyle\int_0^1 \int_{e^y}^{e} \frac{e-x}{\ln x} \mathrm{d}x\,\mathrm{d}y$

   (b) $\displaystyle\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y} \mathrm{d}y\,\mathrm{d}x$

   (c) $\displaystyle\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin(2z)}{4-z} \mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

2. Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2+y^2=4$ and the plane $z+y=3$.

3. A farmer has to fence off a rectangular portion of land along a river, but no fence is needed along the river bank. What should be the dimensions of the fenced region that require the least amount of fencing but enclose a total area of $3,200\,m^2$?

4. Certain bacteria (e.g., *lactobacillus subtilis*), have roughly the shape of a round cylinder of length L and radius R, with two spherical caps of radius R attached to each end. Assuming that it is an evolutionary advantage for this bacteria to have minimal surface area given its fixed volume $v_0 > 0$, find the implied relation between L and R that should be observed in nature.

5. Use polar coordinates to compute the following double integrals:

   (a) $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} \mathrm{d}y\,\mathrm{d}x$

   (b) $\displaystyle\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(1+x^2+y^2) \mathrm{d}x\,\mathrm{d}y$