HW 3 Solutions

. $\vec{r}''(t) = (t-\sin t, 2, t^2)$

$r'(t) = \int r''(t)dt = (\frac{1}{2}t^2 + \cos t + C_1, 2t + C_2, \frac{1}{3}t^3 + C_3)$

$r'(0) = (1,0,1) = (1 + C_1, C_2, C_3)$

$r'(t) = (\frac{1}{2}t^2 + \cos t, 2t, \frac{1}{3}t^3 + 1)$

$r(t) = \int r'(t)dt = (\frac{1}{6}t^3 + \sin t + C_4, t^2 + C_5, \frac{1}{12}t^4 + t + C_6)$

$r(0) = (1,-1,0) = (C_4, C_5, C_6)$

$\therefore r(t) = (\frac{1}{6}t^3 + \sin t + 1, t^2 - 1, \frac{1}{12}t^4 + t)$

.. $r(t) = (\sin t, \sin(2t), \cos(t))$

a. $r'(t) = (\cos t, 2\cos(2t), -\sin t)$

). $r''(t) = (-\sin t, -4\sin(2t), -\cos t)$

: $\int r(t)dt = (-\cos t + C_1, -\frac{1}{2}\cos(2t) + C_2, \sin(t) + C_3)$

l. $\iint (r(t)dt)dt = (-\sin(t) + C_1 t + C_4, -\frac{1}{4}\sin(2t) + C_2 t + C_5, -\cos(t) + C_3 t + C_6)$

). $\alpha(t) = (t+1, t^2, -t)$ $\quad \beta(t) = (\cos t, \sin t, e^{2t} - 1)$ $\quad \alpha(0) = \beta(0) = (1,0,0)$

$\alpha'(t) = (1, 2t, -1)$ $\quad \beta'(t) = (-\sin t, \cos t, 2e^{2t})$

$\alpha'(0) = (1,0,-1)$ $\quad \beta'(0) = (0,1,2)$

$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = (1, -2, 1)$

$x - 2y + z = D$

$1 - 2(0) + 0 = D$

$\therefore x - 2y + z = 1$

. $r(t) = (\sin(\pi t), \cos(\pi t), 10 - t)$

$r'(t) = (\pi\cos(\pi t), -\pi\sin(\pi t), -1)$

$r(5) = (0, -1, 5)$

$r'(5) = (-\pi, 0, -1)$

$L(t) = (0,-1,5) + (-\pi, 0, -1)t$

$z = 0 = 5 - t$

$t = 5$

$L(5) = (-5\pi, -1, 0)$

$\therefore P(-5\pi, -1, 0)$

5. $r(t) = (2t^2, \sqrt{3}t^4, t^6)$

$r'(t) = (4t, 4\sqrt{3}t^3, 6t^5)$

$L = \int_0^2 \sqrt{16t^2 + 48t^6 + 36t^{10}}\, dt =$

$= \int_0^2 2t\sqrt{4 + 12t^4 + 9t^8}\, dt = \int_0^2 2t\sqrt{(3t^4+2)^2}\, dt$

$= \int_0^2 2t(3t^4 + 2)dt = t^6 + 2t^2 \Big|_0^2 =$

$\therefore L = 72$