HW5

1. (a) $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2-y^2} = \frac{(x^2+y^2)(x^2-y^2)}{x^2-y^2}$

$\lim_{(x,y)\to(0,0)} x^2+y^2 = 0$

(b) $\lim_{(x,y)\to(0,0)} \frac{x-y}{x^2+y^2} = \frac{0}{0}$ IND

$y=kx$: $\lim_{x\to 0} \frac{x-kx}{x^2+k^2x^2} = \frac{1-k}{x(1+k)} \Rightarrow$ DNE $\pm\infty$

(c) $\lim_{(x,y)\to(0,0)} \frac{x+y}{\sqrt{x^2+y^2}} = \frac{0}{0}$ IND

$y=kx$: $\lim_{x\to 0} \frac{x(1+k)}{\sqrt{x^2+k^2x^2}} = \frac{x(1+k)}{|x|\sqrt{1+k^2}} = \pm\frac{1+k}{\sqrt{1+k^2}}$ DNE

(d) $\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\sin(x^2+y^2)}{x^4+x^4} = \frac{0}{0}$ IND

$y=kx^2$: $\lim_{x\to 0} \frac{(x^2+kx^2)\sin(x^2+kx^2)}{x^4+k^2x^4} = \frac{\sin(x^2+kx^2)(1+k)}{x^2+k^2x^2}$

$\lim_{x\to 0} \frac{2(1+k)x(1+k)\cos(x^2+kx^2)}{2x(1+k^2)} = \frac{(1+k)^2}{1+k^2}$ DNE

2. $f(x,y,z) = x^2y + \sin(z^2-x)$

$\frac{\partial f}{\partial x} = 2xy + -\cos(z^2-x)$ $\quad \frac{\partial^2 f}{\partial x^2} = 2y + \sin(z^2-x)$ $\quad \frac{\partial^2 f}{\partial x \partial y} = 2x$

$\frac{\partial f}{\partial y} = x^2$ $\quad \frac{\partial^2 f}{\partial y^2} = 0$ $\quad \frac{\partial^2 f}{\partial x \partial z} = 2z\sin(z^2-x)$

$\frac{\partial f}{\partial z} = 2z\cos(z^2-x)$ $\quad \frac{\partial^2 f}{\partial z^2} = 2\cos(z^2-x) - 4z^2\cos(z^2-x)$ $\quad \frac{\partial^2 f}{\partial y \partial z} = 0$

3. $e^z + z = x^2 + y^2$ @ $(1,1,1)$

$3z^2\left(\frac{\partial z}{\partial x}\right) + \left(\frac{\partial z}{\partial x}\right) = 2x$

$\frac{\partial z}{\partial x} = \frac{2x}{3z^2+1}$ @ $(1,1,1)$ $= \frac{2}{4} = \frac{1}{2}$

4. $x(t) = e^{-t}\cos t \quad y(t) = e^{-t}\sin t$

$f(x,y) = x+y$

$w'(t) = \frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = 1(-e^{-t}\cos t - e^{-t}\sin t) + 1(-e^{-t}\sin t + e^{-t}\cos t) =$

$= -2e^{-t}\sin t$

$w(t) = e^{-t}\cos t + e^{-t}\sin t$

$w'(t) = -2e^{-t}\sin t$

5. $V(a,b,c) = abc$; $a(t), b(t), c(t)$

$\frac{dV}{dt} = \frac{\partial V}{\partial a}\frac{da}{dt} + \frac{\partial V}{\partial b}\frac{db}{dt} + \frac{\partial V}{\partial c}\frac{dc}{dt} = (2)(3)(-5) + (1)(3)(-1) + (1)(2)(-3) =$

$\frac{\partial V}{\partial a} = bc \quad \frac{\partial V}{\partial b} = ac \quad \frac{\partial V}{\partial c} = ab$

$\frac{dV}{dt} = -12\ cm^3/s$

$S(a,b,c) = 2ab + 2ac + 2bc$

$\frac{dS}{dt} = \frac{\partial S}{\partial a}\frac{da}{dt} + \frac{\partial S}{\partial b}\frac{db}{dt} + \frac{\partial S}{\partial c}\frac{dc}{dt} = [2(2)+2(3)](-5) + [2(1)+2(3)](-1) + [2(2)+2(1)](-3) =$

$\frac{\partial S}{\partial a} = 2b+2c \quad \frac{\partial S}{\partial b} = 2a+2c \quad \frac{\partial S}{\partial c} = 2b+2a$

$= -5 - 8 - 18 =$

$\frac{dS}{dt} = -31\ cm^3/s$