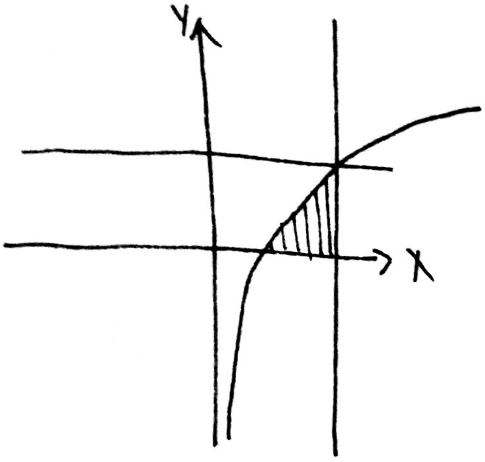
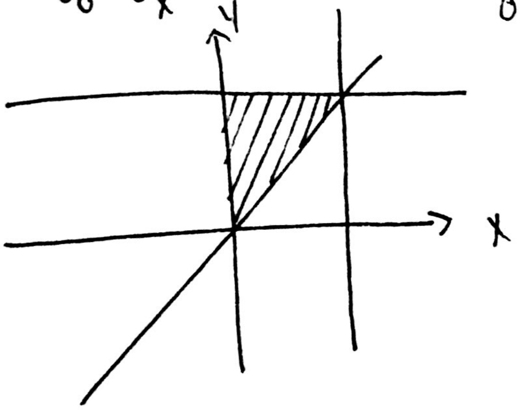
Homework 7

1. a) $\displaystyle\int_0^1\int_{e^y}^{e} \frac{e-x}{\ln x}\,dx\,dy = \int_1^e\int_0^{\ln x}\frac{e-x}{\ln x}\,dy\,dx = \int_1^e (e-x)\,dx = ex-\tfrac{1}{2}x^2\Big|_1^e =$

$$= e^2-\tfrac{1}{2}e^2-e+\tfrac{1}{2} = \tfrac{1}{2}e^2-e+\tfrac{1}{2}$$



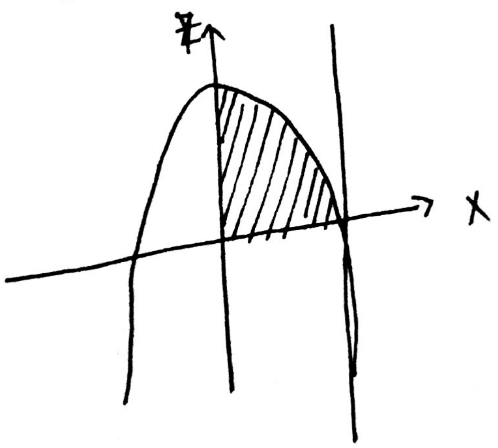
b) $\displaystyle\int_0^{\pi/2}\int_x^{\pi/2}\frac{\sin y}{y}\,dy\,dx = \int_0^{\pi/2}\int_0^{y}\frac{\sin y}{y}\,dx\,dy = \int_0^{\pi/2}\sin y\,dy = -\cos y\Big|_0^{\frac{\pi}{2}} = 1$



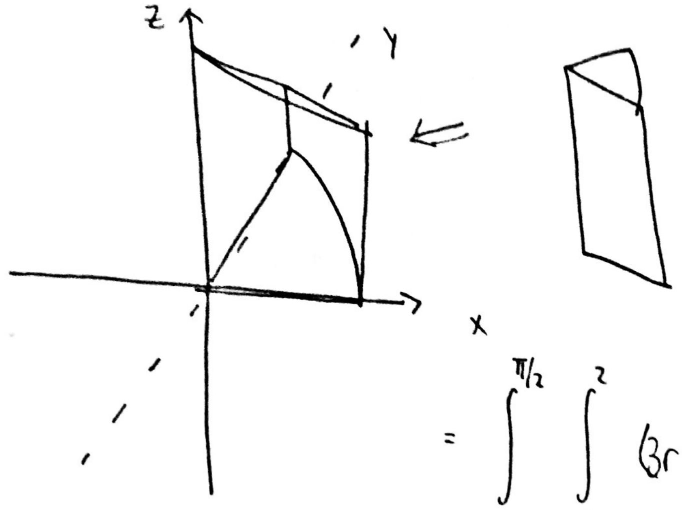
c) $\displaystyle\int_0^2\int_0^{4-x^2}\int_0^x\frac{\sin(2z)}{4-z}\,dy\,dz\,dx = \int_0^2\int_0^{4-x^2}\frac{x\sin(2z)}{4-z}\,dz\,dx = \int_0^4\int_0^{\sqrt{4-z}}\frac{x\sin(2z)}{4-z}\,dx\,dz =$

$$=\frac{1}{2}\int_0^4\sin(2z)\,dz = \frac{-1}{4}\cos(2z)\Big|_0^4 = -\frac{1}{4}\left(\cos 8-\cos(0)\right) =$$

$$= \frac{-1}{4}\left(\cos(8)-1\right)$$



2. $x^2+y^2=4$; $z+y=3$; $z,x,y \geq 0$



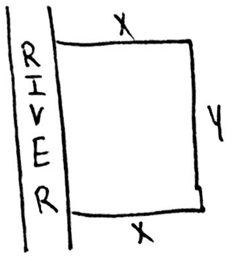
$$V = \int_0^{\frac{\pi}{2}}\int_0^2 [(3-y)-0]\,r\,dr\,d\theta =$$

$$= \int_0^{\frac{\pi}{2}}\int_0^2 (3-r\sin\theta)\,r\,dr\,d\theta =$$

$$= \int_0^{\pi/2}\int_0^2 (3r - r^2\sin\theta)\,dr\,d\theta = \int_0^{\pi/2} \left(\frac{3}{2}r^2 - \frac{1}{3}r^3\sin\theta\right)\Big|_0^2 \,d\theta =$$

$$= \int_0^{\pi/2} \left(6 - \frac{8}{3}\sin\theta\right)d\theta = 6\theta + \frac{8}{3}\cos\theta \Big|_0^{\frac{\pi}{2}} = 3\pi - \frac{8}{3}$$

3.



$f(x,y) = 2x+y$; $x,y > 0$

$g(x,y) = xy - 3200$

$\nabla f = \lambda \nabla g \Rightarrow (2,1) = \lambda(y,x)$

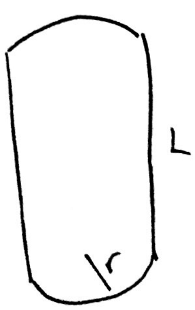
$$\frac{2}{y} = \frac{1}{x}$$

$$2x = y$$

$$3200 = xy = 2x^2$$

$x = 40m$; $y = 80m$

4.



~~area~~ $f(r,L) = 2\pi rL + 4\pi r^2$ (SA) $\quad r>0$; $L \geq 0$

$g(r,L) = \pi r^2 L + \frac{4}{3}\pi r^3 - V_0$ (V)

$\nabla f = \lambda \nabla g$

$(2\pi L + 8\pi r,\ 2\pi r) = \lambda(2\pi rL + 4\pi r^2, \pi r^2)$

$$\frac{2\pi L + 8\pi r}{2\pi rL + 4\pi r^2} = \frac{2\pi r}{\pi r^2}$$

$$\frac{L+4r}{rL+2r^2} = \frac{2}{r}$$

$$0 = rl \Rightarrow 1 = 0$$

$\therefore$ If $r, l > 0$, minimum SA if

$r >> L$