Homework Set 1

Due: Sep 8, 2015 (in class)

1. Find the most general solution $y(x)$ to each of the following linear ODEs:

$$y' + 4y = 0 \qquad y' - 4y = 0 \qquad xy' + 3x^3 y = 0$$
$$y'' + 4y = 0 \qquad y'' - 4y = 0 \qquad y'' - 6y' + 9y = 0$$

2. Write a formula for the solution $y(t)$ of $y'' + y' + \frac{1}{4}\lambda y = 0$ in each of the cases $\lambda < 1$, $\lambda = 1$ and $\lambda > 1$. Show that if $\lambda \geq 1$, then $\lim_{t\to+\infty} y(t) = 0$.

3. What is the dimension n of the space of solutions to the linear ODE $u''' + 9u' = 0$? Find n functions that give a basis of this vector space, and use them to write a formula for the general solution $u(t)$.

4. Say $w(t)$ satisfies the differential equation

$$aw'' + bw' + cw = 0, \tag{1}$$

   where $a > 0$, $b \geq 0$, and $c > 0$. Let $E(t) = \frac{1}{2}aw'^2 + \frac{1}{2}cw^2$.
   a) Without solving the differential equation, show that $E'(t) \leq 0$.
   b) Use this to show that if $w(0) = 0$ and $w'(0) = 0$, then $w(t) = 0$ for all $t \geq 0$.
   c) Suppose that $u(t)$ and $v(t)$ both satisfy equation (1), and also $u(0) = v(0)$ and $u'(0) = v'(0)$. Show that $u(t) = v(t)$ for all $t \geq 0$.

5. Say $u(x,t)$ has the property that $\dfrac{\partial u}{\partial t} = 3u$ for all points $(x,t) \in \mathbb{R}^2$.
   a) Find some such function – other than the trivial $u(x,t) \equiv 0$.
   b) Find the most general such function.
   c) If $u(x,t)$ also satisfies the initial condition $u(x,0) = \sin 3x$, find $u(x,t)$.

6. Haberman 1.2.3

7. Haberman 1.4.1 (a), (b), (c), (e), (f)

8. Haberman 1.4.7

9. Haberman 1.4.10

10. Haberman 1.4.11